03/10/20 Introduction to Logarithms

What are logarithms and why do we need to study them?
Logarithms appear in all sorts of calculations in engineering and science, business and economics. The Richter scale measurement for earthquakes is based upon logarithms as is the decibel scale in acoustics. Logarithms were originally developed to simplify complex arithmetic calculations. Before the days of calculators, they were used to assist in the process of multiplication by replacing the operations of multiplication by addition. Similarly, they enabled the operation of division to be replaced by subtraction.

Today of course, we have calculators and scientific notation to deal with such large numbers. So, at first glance, it would seem that logarithms have become obsolete. However, they are still important in helping to solve exponential equations. I will give you the formal definition of a logarithm in just a second, but what you need to realize is that logarithms ALWAYS relate back to exponential equations.

FORMAL DEFINITION OF LOGARITHM:

**** Suppose $b > 0$ and b is not equal to 1, then $\log_b y = x$ if and only if $b^x = y$.

With logs, we read this as: "the log base b of y is equal to x".

You must be able to convert an exponential equation into logarithmic form and vice versa, so let's get some practice!!

Write each equation in logarithmic form:

Ex 1: $2^3 = 8$

$\log_2 8 = 3$

Ex 2: $2^{-3} = \frac{1}{8}$

$\log_2 \frac{1}{8} = -3$

Ex 3: $5^0 = 1$

$\log_5 1 = 0$

Ex 4: $10^{-1} = \frac{1}{10}$

$\log_{10} \frac{1}{10} = -1$

Ex 5: $81^{\frac{1}{2}} = 9$

$\log_{81} 9 = \frac{1}{2}$

Ex 6: $4^{\frac{3}{2}} = 8$

$\log_4 8 = \frac{3}{2}$

Write each equation in exponential form:

Ex 7: $\log_3 81 = 4$

$3^4 = 81$

Ex 8: $\log_2 \frac{1}{8} = -3$

$2^{-3} = \frac{1}{8}$

Ex 9: $\log_{10} 100 = 2$

$10^2 = 100$

Ex 10: $\log_5 \frac{1}{125} = -3$

$5^{-3} = \frac{1}{125}$

Ex 11: $\log_{27} 3 = \frac{1}{3}$

$27^{1/3} = 3$

***Remember: The logarithm tells us what the exponent is!

exponent

$2^3 = 8 \quad \leftrightarrow \quad \log_2(8) = 3$

base

$a^x = y$

$\log_a(y) = x$

A logarithm is an exponent. Logarithms "undo" exponents – just like division "undoes" multiplication and addition and subtraction "undo" each other! Here is a convenient rule to remember: When working with logarithms, if ever you get "stuck", try rewriting the problem in exponential form. Conversely, when working with exponential expressions, if you ever get "stuck", try rewriting the problem in logarithmic form.

Let's see if this simple rule can help us solve some of the following problems:

Ex 12: $\log_5 125 = x$

$5^x = 125$

$5^x = 5^3$ so $\boxed{x=3}$

Ex 13: $\log_3 27 = x$

$3^x = 27$

$3^x = 3^3$ $\boxed{x=3}$

Ex 14: $\log_5 1 = x$

$5^x = 1$

$5^x = 5^0$ $\boxed{x=0}$

Ex 15: $\log_6 \sqrt{6} = x$

$6^x = \sqrt{6}$

$6^x = 6^{1/2}$ $\boxed{x = 1/2}$

Ex 16: $\log_2 2^{\frac{1}{3}} = x$

$2^x = 2^{1/3}$

so $\boxed{x = 1/3}$

Ex 17: $\log_{32} 2 = x$

$32^x = 2$

$2^{5x} = 2^1$

$5x = 1$

$\boxed{x = \frac{1}{5}}$

Ex 18: $\log_{27} 9 = x$

$27^x = 9$

$3^{3x} = 3^2$

$3x = 2$

$\boxed{x = 2/3}$

Ex 19: $\log_3 \frac{1}{9} = x$

$3^x = \frac{1}{9}$

$3^x = 3^{-2}$ $\boxed{x=-2}$

Ex 20: $\log_{\frac{1}{7}} 343 = x$

$\frac{1}{7}^x = 343$

$7^{-1x} = 7^3$

$-x = 3$

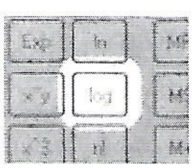
$\boxed{x=-3}$

Common Logarithms: Base 10

Sometimes a logarithm is written **without** a base, like this:

log(100)

This ***usually*** means that the base is really 10.



It is called a "common logarithm". Engineers love to use it.

On a calculator it is the "log" button.

It is how many times we need to use 10 in a multiplication, to get our desired number.

Example: $\log 1000 = \log_{10} 1000 = 3$

Find the following without a calculator: $\log 10000 \rightarrow \log_{10} 10000 = x$

$10^x = 10000$

$10^x = 10^4$

$\boxed{x=4}$